Radial Basis Functions for Topology Optimisation of Engineering Structures

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***Abstract*—Truss topology optimisation is a well-established field in structural engineering, traditionally focusing on determining optimal node locations and member sizing to minimise compliance or weight under prescribed loads. Recent advances have explored more flexible design representations to overcome limitations of fixed-topology approaches. In this context, Radial Basis Functions (RBFs) offer a tool for topology optimisation, providing smooth, continuous control over geometry and structural parameters. By incorporating RBFs, optimisation algorithms can efficiently generate diverse structural solutions, enable finer control over cross-sectional areas, and improve convergence rates. This work highlights the potential of RBF-based representations to enhance truss topology optimisation, offering designers a versatile and computationally efficient framework for exploring complex, high-performance structural configurations. The optimisation of a car chassis subjected to multiple load cases is used as a case study to demonstrate the capabilities of the approach.**

***Index Terms*—topology optimisation, evolutionary computation, radial basis functions, engineering design**

I. Introduction

Truss structures are widely used in engineering problems because they achieve high strength and stiffness with minimal material by aligning structural members with load paths. From a design perspective, truss systems are also characterised by a large number of design variables such as cross-sectional areas, number of vertices and vertex placement, while exhibiting a relatively small number of state or output variables, including nodal displacements [1]. This imbalance between design and output variables makes truss structures particularly attractive, yet challenging, targets for structural optimisation. As a consequence, topology optimisation for truss structures has been extensively explored in structural engineering, with most approaches focusing on adjusting node positions or displacements to minimise compliance or weight [1]. Many existing optimisation methods also rely on predefined ground structures, which constrain the range of topological transformations that can be achieved and limit the discovery of more diverse or efficient structural configurations [2], [3].

Evolutionary algorithms (EAs) have been widely applied to topology optimisation, offering a flexible search process capable of navigating complex, non-convex design spaces. Their ability to evolve structural layouts without explicit gradients makes them well suited to problems where topology, geometry, and sizing interact in non-linear ways. A wide range of structural representations has therefore been explored in the literature in conjunction with EAs [4]; however, relatively little attention has been given to representations that can simultaneously optimise multiple structural attributes within a compact encoding, limiting the efficient co-evolution of geometry and sizing.

In this paper, Radial Basis Functions (RBFs) [5] are introduced as a representation for truss topology optimisation. This approach enables the simultaneous optimisation of multiple structural attributes, such as node placement and cross-sectional variation, through a unified, compact encoding, expanding the space of admissible designs beyond fixed ground-structure formulations. An EA is employed to optimise the RBF parameters, enabling the evolution of multiple spatial fields, each of which defines a distinct structural attribute within the resulting design.

To demonstrate the capabilities of this approach, the optimisation of a 2D car chassis is used as a case study. Distinct load cases are examined to illustrate how different types of RBFs adapt structural topology and sizing in response to varying loading conditions.

The contributions of this paper are the following:

1) A compact RBF aggregation representation for truss topology with a relatively low number of design variables is introduced, enabling fast convergence and efficient exploration of the design space under an evolutionary algorithm.
2) Different types of RBFs can be combined to optimise multiple attributes of a structure, as demonstrated, including node placement, Young's modulus and cross-sectional areas.
3) The flexibility of the approach on multiple load cases relevant to automotive chassis design is demonstrated.

The remainder of this paper is organised as follows. Section II reviews relevant work on truss optimisation and the use of RBFs in related domains. Section III outlines the proposed methodology. Section IV presents the experimental results, and Section V discusses the limitations of the approach and potential directions for future work.

II. Literature review

Truss optimisation has been extensively studied in structural engineering, with classical formulations focusing on determining optimal node locations and member sizing to minimise compliance or weight under prescribed loading conditions [1], [6], [7]. Although these approaches are effective within fixed-topology frameworks [8]–[10], the explicit addition of nodes during optimisation has received comparatively little attention [11], [12], limiting the structural diversity that can be explored.

Evolutionary algorithms (EAs) have been widely employed for topology optimisation [4], offering the ability to navigate complex, discontinuous design spaces. Several EA-based methods have incorporated mechanisms for structural growth. For example, Dubey et al. [13] use gene regulatory networks (GRN) to determine when an edge should be split into two members connected by a newly introduced node. However, this GRN requires a ground structure to operate upon. Grammar-based approaches have also been explored to generate structures with variable numbers of vertices [14], [15], enabling richer topological transformations. Truss structures can also be represented as graphs and operations can be applied directly to the adjacency matrices. However, this can lead to non-feasible solutions [2], [16].

Radial Basis Functions (RBFs), originally introduced by Hardy [5], have become widely used across computational science due to their strong interpolation capabilities and mathematical flexibility [17]. They have been successfully applied to the approximation of solutions for both ordinary and partial differential equations [18], as well as to more general function approximation tasks in machine learning [19].

Beyond numerical approximation, RBFs have proven valuable in geometric modelling and mesh generation. Early work on RBF-based mesh manipulation [20] demonstrated their memory efficiency and smooth deformation properties. Subsequent studies have used RBFs to control mesh resolution and parameterisation in deformable-object simulations [21], highlighting their versatility for representing both spatial geometry and associated fields.

Despite their adoption in other domains, the use of RBFs in truss topology optimisation remains limited. In particular, existing approaches lack the co-evolution of both geometry and member properties, such as cross-sectional area, within an RBF-based representation. This gap motivates the development of more expressive optimisation frameworks capable of leveraging RBFs to encode continuously varying structural characteristics.

III. Methodology

A. Radial Basis Functions implementation

This paper introduces the use of Radial Basis Functions (RBFs) for topology optimisation. The design space is sampled from a density field generated by an aggregation of RBFs to define the structural properties. The aggregated field is expressed in Equation 1:

$$f(\mathbf{x}) = \sum_{i=1}^{N} w_i \, \phi\left(\|\mathbf{x} - \mathbf{c}_i\|\right) \quad (1)$$

where $\mathbf{x}$ denotes a point in space (2D or 3D), $\mathbf{c}_i$ is the centre of the i-th RBF, w_i is its corresponding weight (amplitude or influence), and $\phi(r)$ is the chosen radial basis function (e.g., linear, Gaussian, sinusoidal or multiquadric). In this work, a linear radial basis function is adopted due to its simplicity and explainability, in which the distance to a basis centre is linearly related to the field intensity, as defined in Equation 2. The parameter ε_i controls the shape, or spatial extent, of the i-th RBF.

$$\phi(\|\mathbf{x} - \mathbf{c}_i\|) = \varepsilon_i \|\mathbf{x} - \mathbf{c}_i\| \quad (2)$$

In this paper, the parameters w_i, ε_i, and $\mathbf{c}_i$ are evolvable. Future work could explore the evolution of different functions ϕ and different operations between RBFs besides aggregation.

Two types of RBFs are employed in this paper, each serving a distinct purpose: (1) to determine the cross-sectional areas of the edges (w-RBF), and (2) to define the number and spatial distribution of additional vertices (v-RBF). However, more types of RBFs can be implemented to optimise other aspects of the structure, such as cross-sectional profile shapes and Young's modulus.

For the first purpose (w-RBF), the midpoint of each edge is evaluated as $\mathbf{x}$ in Equation 1. A higher value of the aggregated field $f(\mathbf{x})$ corresponds to a proportionally larger cross-sectional area (thicker edge).

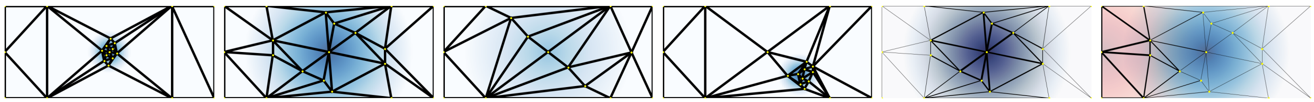
For the second purpose (v-RBF), the number of additional vertices is determined by the aggregated weights w_i. Specifically, a new vertex is instantiated for every 0.1 increment in the global aggregated influence (e.g., a total weight of 0.7 results in 7 vertices). The coordinates of these vertices are governed by the interaction between ε_i and w_i. ε_i defines the support diameter (the spatial range of influence) for each RBF, while w_i acts as the local density intensity, which controls the relative spacing of nodes within that range to maintain an even distribution. Delaunay triangulation is used to link all the vertices together with edges, which form the members of the truss structure.

Figure 1 illustrates how variations in the RBF parameters ε, w, and the centre coordinates (x, y) influence the resulting structural configuration.

B. Application

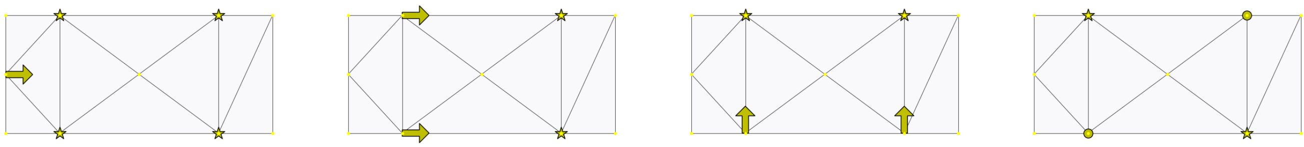
The application of RBFs explored in this study involves the evolution of 2D car chassis structures. In order to demonstrate the flexibility of the RBFs, four different loading conditions are studied, as illustrated in Figure 2: crash, cornering, torsion, and braking. The dimensions of the chassis and location of the wheels are based on the Tesla Model S[1]. The structures are simulated using Calculix [22].

[1]https://www.dimensions.com/element/tesla-model-s



(a) (b) (c) (d) (e) (f)

Fig. 1: Examples of structures generated with different RBF parameter values. Figures (a) and (b) illustrate variations in ε. Figures (b) and (c) show the effect of changing the weight w. Figures (a) and (d) demonstrate modifications to the RBF centre coordinates (x, y). Figures (e) and (f) highlight changes in cross-sectional area, as indicated by the red gradient.



(a) Crash (b) Braking (c) Cornering (d) Torsion

Fig. 2: Load cases explored in this paper: crash (a), braking (b), cornering (c) and torsion (d). Stars represent fixed vertices, arrows represent the direction of load, and circles represent out-of-plane loads.

Two experiments are conducted to evaluate the approach. In the first, only the number and positions of vertices are evolved (v-RBF). In the second, both the vertex configuration (v-RBF) and the cross-sectional areas (w-RBF) of the structural edges are evolved simultaneously. Figure 3 illustrates examples of structures produced with random genomes. The blue gradient represents v-RBFs, and the red gradient represents w-RBF.

A multi-objective evolutionary algorithm (NSGA-II)[2] with its default values is employed to minimise two conflicting objectives: the total structural volume (as a proxy for material cost) and global maximum deflection over all nodes, promoting a balance between lightweight design and stiffness. A total of 10,000 evaluations were performed using a population size of 100 over 100 generations. Constraints are used to prevent the Pareto front arms from extending to unfeasible solutions.

Parameters and values used for the experiments in this paper can be found in Table I.

TABLE I: Parameters used for the experiments shown in this paper.

Genome value range for each RBF	
c_x	$[0, 4970]mm$
c_y	$[-1094.5, 1094.5]mm$
w	$[-1, 1]$
ε	$[49.7, 4970]mm$
EA hyperparameters	
Generations	100
Population	100
Constraint 1	Volume $< 6x10^6 mm^3$
Constraint 2	Deflection $< 250mm$
Replicates	15
Structural properties	
Each load	$1000N$ ($100N$ - torsion)
Cross-sectional area	$(0, 100]mm^2$
RBF types (structural attributes)	
Vertex number and placement	v-RBF
Cross-sectional area	w-RBF

IV. Experiments

A. *Pareto front analysis (quantitative analysis)*

This section examines the behaviour of the evolutionary algorithm (EA) in terms of both convergence properties and the characteristics of the final Pareto fronts. To assess the quality of the obtained fronts, the hyper-volume metric is used, with the reference point positioned at the boundary of the design constraints $[< 6 \times 10^6 \text{ mm}^3, 250 \text{ mm}]$.

The introduction of the w-RBF has a clear influence on the optimisation outcome. Specifically, it shifts the Pareto front leftward along the volume axis by reducing material around the front and rear regions of the car (Figure 4). In most load cases, structural changes outside the central chassis section between the front and the rear wheels are minimal; however, the crash load case remains an exception where the front experiences meaningful deformation pressure.

A notable distinction between the scenarios with and without cross-sectional area regulation is the density of solutions on the Pareto front (Figure 4). When the w-RBF is enabled, more solutions populate the front. This occurs because the v-RBF typically identifies a viable structural configuration, after which the w-RBF produces a family of variants with different

[2]https://pymoo.org/

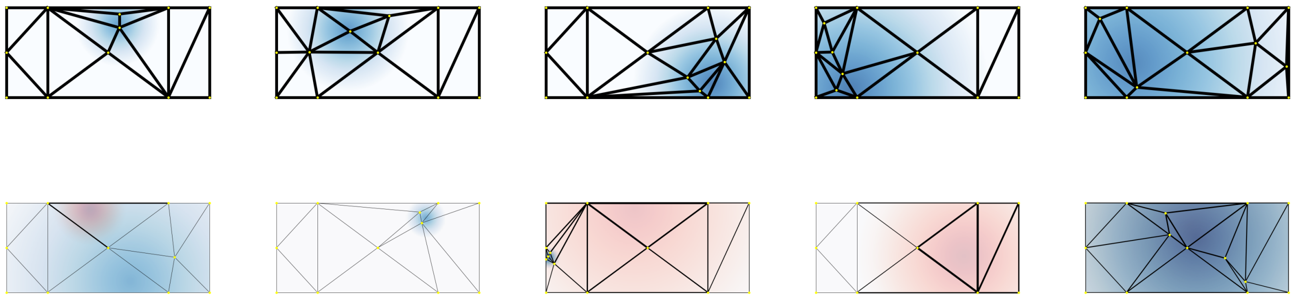


Fig. 3: Examples of randomly generated structures using v-RBFs (top row) and v-RBF + w-RBF (bottom row). The blue gradient represents the intensity and spatial influence of the v-RBFs, while the red gradient illustrates the corresponding effect of the w-RBFs.

cross-sectional area distributions. This expanded set of alternatives is advantageous for designers, offering them greater flexibility when selecting structures for further refinement or downstream engineering tasks.

In terms of convergence behaviour, the RBF-based representation leads to efficient optimisation. Stable solutions are typically reached after approximately 2,000 evaluations for configurations using only v-RBFs, and around 10,000 evaluations when the w-RBF is included (Figure 5). This efficiency is largely attributed to the relatively compact genome representation.

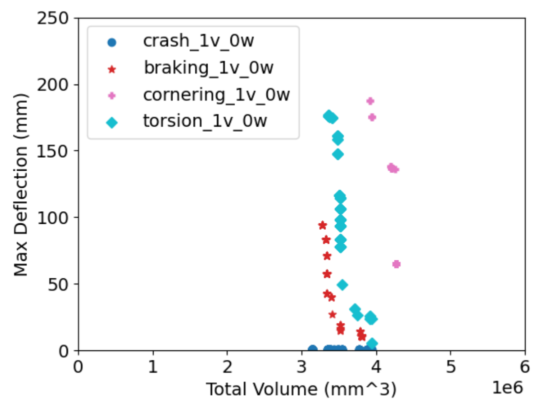
Overall, these results demonstrate that the proposed RBF-based representation enables both efficient optimisation and the generation of diverse, high-quality structural solutions. The combination of v-RBFs and w-RBFs not only improves the exploration of the design space but also provides designers with a broader set of viable alternatives that balance stiffness and material usage. The method therefore offers a practical and computationally effective framework for early-stage structural design, supporting informed decision-making and enabling richer structural variation than traditional topology optimisation approaches.

B. Structural analysis (qualitative analysis)

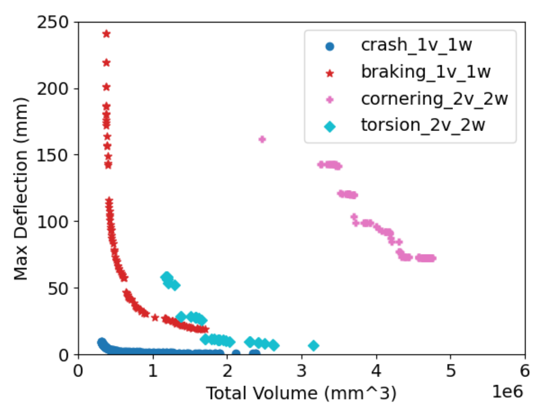
This section provides an engineering-focused interpretation of the evolved structures shown in Figures 6, highlighting how their features relate to the behaviour of the underlying RBFs.

For the cornering load case (Figures 6c and g), the optimisation favours the formation of a dense cluster of nodes near the lower region of the structure, close to the applied forces, and this is shown with the range and intensity of the blue gradient. This clustering increases local stiffness and improves load distribution, while the thinner edges in the lower region contrast with the thicker members at the top, where forces accumulate.

The evolved structures for the torsion load case are largely similar with and without the inclusion of the w-RBF (Figures 6d and h). The main differences appear in the thinner front

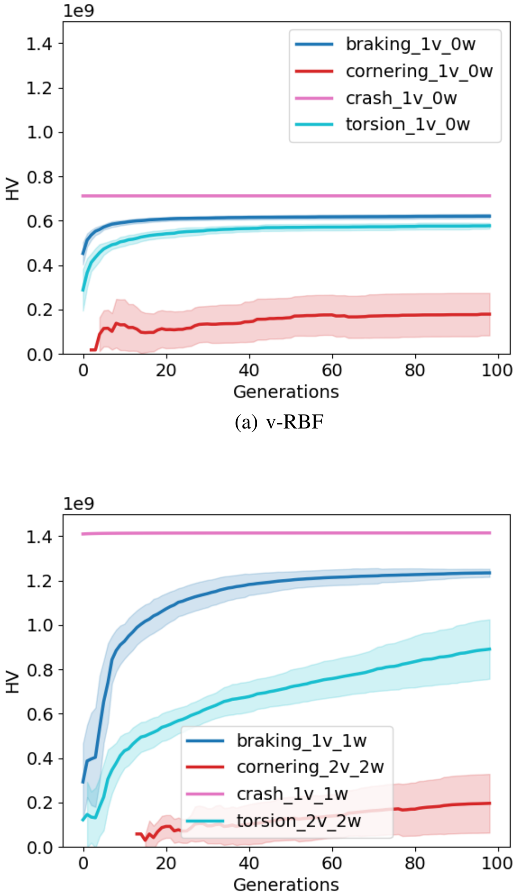


(a) v-RBF



(b) v-RBF + w-RBF

Fig. 4: Best Pareto fronts across all replicates for v-RBF (a) and v-RBF + w-RBF (b). The addition of the w-RBF shifts the Pareto fronts leftward, reducing the overall structural weight. The number represents the number of RBFs used for each type.



(a) v-RBF

(b) v-RBF + w-RBF

Fig. 5: Convergence graphs. The RBF-based optimisation converges relatively quickly, reaching stable solutions within roughly 2,000 evaluations for v-RBFs and around 10,000 evaluations when the w-RBF is included. The number represents the number of RBFs used for each type.

and rear edges, illustrating how the algorithm reduces volume in areas that experience lower stresses, consistent with the behaviour discussed previously. This is shown with the red gradient that extends up to where the central chassis section ends, making all the edges in the central chassis section thicker.

In the crash load scenario using v-RBFs (Figure 6a), a distinctive horizontal edge emerges between the central vertex and the rear support forming a load path. This feature plays a key role in distributing horizontal loads more effectively across the four support points, improving the structure's ability to accommodate the sudden unidirectional force.

Overall, the evolved structures demonstrate a clear correspondence between local RBF influence and emergent structural behaviour. The v-RBFs primarily define the spatial distribution of nodes and member density, while the w-RBF refines member thickness based on load intensity. Together, these mechanisms allow the optimisation process to create structurally efficient, load-aware geometries that remain interpretable from an engineering standpoint.

V. Discussion

The objective of this paper is to demonstrate how RBFs evolve structures under different individual load cases. However, the next natural step is to combine these load cases to generate more realistic structures that can withstand multiple loading conditions rather than a single one. This becomes more important when special attention is required, as described next.

A common characteristic when evolving the cross-sectional area is that the edges become thinner at the rear of the car, and in some cases also at the front, across most of the load cases explored in this paper. However, with such low cross-sectional areas, the car's boot would not be able to carry heavy items, and the structure would likely fail in a rear-end collision. Future work should therefore include load cases applied to, other regions; otherwise, the boot will effectively be removed by the optimisation process.

In this paper, a single load case is optimised at a time. Future work should also consider combining and jointly optimising multiple load cases to investigate the evolution of the RBFs under these conditions, as well as the emergence of symmetry and novel structures capable of withstanding multiple loading scenarios.

The number and types of RBFs used in this study were fixed; future work could extend the framework by evolving these choices as part of the optimisation process. Additional structural attributes, such as cross-sectional profile shapes and the Young's modulus of members, could also be incorporated into the representation to enable impact absorption.

Despite these limitations, the method offers a flexible foundation for more advanced multi-physics or multi-material structural optimisation.

VI. Conclusions

This paper introduced a novel RBF-based representation for truss topology optimisation that enables the simultaneous evolution of different attributes of the topology, including node placement and cross-sectional areas, through a compact and expressive genome.

Results demonstrate structurally diverse and load-aware geometries while maintaining fast convergence. Experiments on four automotive load cases demonstrate that the RBF-based approach yields richer Pareto fronts, improved material efficiency, and interpretable structural variations directly linked to the underlying RBF influence fields. These results highlight the potential of RBFs as a powerful alternative to traditional ground-structure or discrete topology-operator methods.

Future work will extend the approach to multi-load optimisation, incorporate manufacturability constraints, and explore 3D representations for more complex engineering structures.

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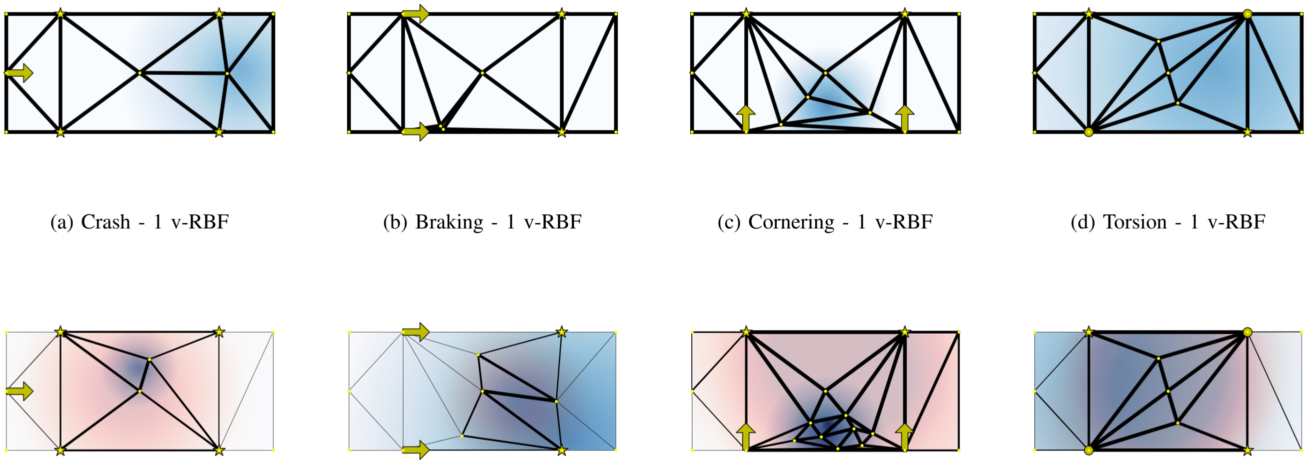

(a) Crash - 1 v-RBF
(b) Braking - 1 v-RBF
(c) Cornering - 1 v-RBF
(d) Torsion - 1 v-RBF
(e) Crash - 1 v-RBF + 1 w-RBF
(f) Braking - 1 v-RBF + 1 w-RBF
(g) Cornering - 2 v-RBF + 2 w-RBF
(h) Torsion - 2 v-RBF + 2 w-RBF

Fig. 6: Examples of evolved solutions taken from the final Pareto front. The blue gradient represents v-RBF and the red gradient represents w-RBF. The number represents the number of RBFs used for each type.

sign”[3]. The authors would like to acknowledge the Institute for Safe Autonomy (ISA)[4].

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